NEUTRALIZATION OF A SPHERICALLY DIVERGING CURRENT OF CHARGED PARTICLES

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The process of neutralizing a spherically diverging ion current is considered. It is shown that two different types of behavior are possible for the potential in the neutralized flow depending upon the values of the characteristic parameters. In contrast to the plane characterised by undamped potential oscillations [1–4], in the spherical case, the potential either performs damped oscillations or montonically approaches a limiting value. As an example, numerical results are given for the case of a cesium ion current neutralized by electrons.

Many articles have been devoted to the problem of neutralizing ion beams. These deal in detail with the neutralization process in a one-dimensional flat ion beam, where all the parameters depend only upon the distance to some plane. The solutions show that undamped potential oscillations occur if the dissipative processes are ignored [1–4]. Actually, beams of charged particles, if not subjected to additional focusing, diverge beyond the accelerating electrode. Therefore the problem of neutralizing a spherically diverging flow is of interest, since it approximates more closely the form of actual ion beams.

Figure 1 shows the electrode arrangement used to obtain a spherically diverging flow. The first electrode of radius r^0 is the ion emitter, the second, a grid of radius r_0, is the accelerating electrode, and the third, a neutralizing grid, is the source of oppositely charged particles. The ion current is created by the potential difference Φ_0. Owing to the energy acquired in the accelerating gap 1, the ions reach the neutralizing grid, which is at a potential $0 \le \Phi_* < \Phi_0$, and then enter the exterior region 3. The electric field of the ion beam extracts oppositely charged particles from the neutralizer; this flux then propagates in the exterior space. The neutralizer is assumed to be ideal, i.e., there is no limit to its emitting capacity. The problem of accelerating the beam in the accelerating gap was considered for the spherical case in [5] where the ion current is calculated as a function of the potential Φ_0 and the length of the accelerating gap.

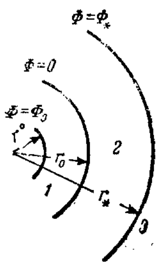

Fig. 1

The problem need not be solved for region 2, since the distance $r_* - r_0$ can always be chosen so that the entire ion flux reaches the neutralizer. Therefore we shall only consider the neutralization process in the exterior region.

The behavior of such a mixed flow is described by the system of equations

$$mu^2 = 2e(\Phi - \Phi_*), \qquad 4\pi r^2 nu = J$$
$$Mu_i^2 = 2e(\Phi_0 - \Phi), \qquad 4\pi r^2 n_i u_i = J_i \tag{1}$$
$$\frac{d}{dr} r^2 \frac{d\Phi}{dr} = 4\pi e r^2 (n - n_i).$$

Here M, u_i, and n_i are, respectively, the ion mass, velocity, and density; m, u, and n are, respectively, the mass, velocity, and density of the oppositely charged particles; e is the electron charge (the particles are considered to be single-charged); Φ is the potential; and J_i, J are the total fluxes of the corresponding particles.

In deriving this system it was assumed for simplicity that the particles leave the neutralizer with zero velocity.

By eliminating the densities from the Poisson equation, we obtain an equation for the potential:

$$\frac{d}{dr} r^2 \frac{d\Phi}{dr} = e\left(\frac{J\sqrt{{}^1/_2\, m/e}}{\sqrt{\Phi - \Phi_*}} - \frac{J_i \sqrt{{}^1/_2\, M/e}}{\sqrt{\Phi_0 - \Phi}}\right). \tag{2}$$

The boundary conditions will be

$$\Phi = \Phi_*, \qquad \frac{d\Phi}{dr} = 0 \quad \text{at} \quad r = r_*. \tag{3}$$

Zero electric field intensity means that the current extracted from the neutralizer is limited by the space charge. This is a consequence of the assumption of unlimited emission from the neutralizer.

Introducing the dimensionless variables

$$\varphi = \frac{\Phi - \Phi_*}{\Phi_0 - \Phi_*}, \qquad r = r_* e^s, \tag{4}$$

we obtain the basic equation

$$\frac{d^2\varphi}{ds^2} + \frac{d\varphi}{ds} = A\left(\frac{\alpha}{\sqrt{\varphi}} - \frac{1}{\sqrt{1-\varphi}}\right),$$
$$\varphi = \frac{d\varphi}{ds} = 0 \quad \text{for} \quad s = 0, \tag{5}$$

where

$$\alpha = \frac{J}{J_i}\frac{\sqrt{m}}{\sqrt{M}}, \qquad A = \frac{J_i\sqrt{Me}}{\sqrt{2}\,(\Phi_0 - \Phi_*)^{3/2}}. \tag{6}$$

In the plane of the hodograph for φ and $\phi = d\varphi/ds$ obtained the equation is equivalent to the system

$$\frac{d\varphi}{ds} = \psi, \qquad \frac{d\psi}{ds} = -\psi + A\left(\frac{\alpha}{\sqrt{\varphi}} - \frac{1}{\sqrt{1-\varphi}}\right). \tag{7}$$

This system does not have a periodic solution, since it satisfies the Bendixon criterion [6]. A singular point of the system is

$$\varphi = \alpha^2/(1 + \alpha^2), \qquad \psi = 0.$$

This point corresponds to an infinitely distant point. In order to establish the nature of the singular point, we expand the right side of system (7) in a series in the neighborhood of this point and, discarding terms of second order of smallness and above, we obtain the equation

$$\frac{d\psi}{d\varphi_1} = -\frac{\lambda^2\varphi_1 + 4\psi}{4\psi} \tag{8}$$

$$\left(\varphi = \frac{\alpha^2}{1+\alpha^2} + \varphi_1, \qquad \lambda^2 = \frac{2A\,(1+\alpha^2)^{5/2}}{\alpha^2}\right).$$

The roots of its characteristic equation $k^2 + k + {}^1/_4\lambda^2 = 0$ are equal to

$$k_{1,2} = -{}^1/_2 \pm {}^1/_2\sqrt{1 - \lambda^2}.$$

Two cases are clearly possible.

1) For $\lambda_2 \le 1$. In this case, the singular point is a "node." The integral curves pass through this point.

2) For $\lambda^2 > 1$. In this case the roots $k_{1,2}$ are complex with a negative real part. The singular point is a stable "focus." The integral curves in the phase plane coil about this point.

In the real plane, in case (1) the potential does not oscillate, but monotonely increases from zero to its limiting value $\varphi_\infty = \alpha^2 (1+\alpha^2)^{-1}$. Case (2) corresponds to damping potential oscillations near the limiting value.

The behavior of the potential at large values of the parameter s is described by the asymptotic formulas

$$\varphi = \varphi_\infty + c_1 \exp\left[-{}^1/_2\,(1+\sqrt{1-\lambda^2})s\right] + c_2 \exp\left[-{}^1/_2\,(1-\sqrt{1-\lambda^2})s\right] \quad \text{for} \quad \lambda^2 < 1, \qquad (9)$$

$$\varphi = \varphi_\infty + c_3 e^{-{}^1/_2 s} + c_4 s e^{-{}^1/_2 s} \quad \text{for} \quad \lambda^2 = 1, \qquad (10)$$

$$\varphi = \varphi_\infty + e^{-{}^1/_2 s}\left(c_5 \sin {}^1/_2 \sqrt{\lambda^2 - 1}\, s + c_6 \cos {}^1/_2 \sqrt{\lambda^2 - 1}\, s\right) \quad \text{for} \quad \lambda^2 > 1. \qquad (11)$$

At small values of the parameter s, the solution to equation (5) with corresponding boundary conditions has the form

$$\varphi = ({}^9/_4 A\alpha)^{2/3} s^{4/3} \sum_{k=0}^{\infty} a_k s^{1/3 k}. \qquad (12)$$

Below, values are given for the first coefficients in series (12):

$$a_0 = 1, \quad a_1 = 0, \quad a_2 = -{}^1/_5 \alpha^{-1} ({}^9/_4 A\alpha)^{1/3},$$

$$a_3 = -{}^9/_{11}, \quad a_4 = {}^1/_{28} a_2^2, \quad a_5 = -{}^{225}/_{616} a_2, \ldots$$

In the phase plane, the integral curve passing through the origin will be

$$\psi = \varphi^{1/4} \sum_{k=0}^{\infty} b_k \varphi^{1/4 k}. \qquad (13)$$

The coefficients of the expansion are found from the formulas

$$b_0 = 2\sqrt{A\alpha}, \quad b_1 = 0, \quad b_2 = -A / b_0, \quad b_3 = -{}^4/_5$$

$$b_4 = -\frac{A^2}{2b_0^3}, \quad b_5 = -\frac{8A}{35 b_0^2}, \quad b_6 = \frac{4}{25 b_0} - \frac{A^3}{2b_0^5} - \frac{A}{4b_0}.$$

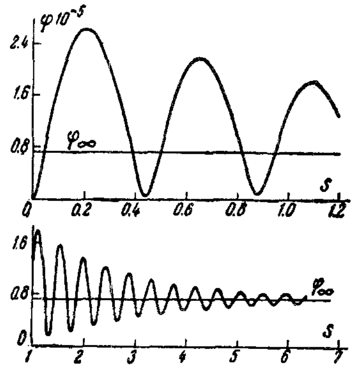

Fig. 2

The rest can be computed from the recurrence relations

$$b_{4m-1} = -\frac{1}{b_0}\left(\frac{4}{4m+1} b_{4m-4} + b_1 b_{4m-2} + \ldots + b_i b_{4m-i-1} + \ldots + b_{2m-1} b_{2m}\right) \qquad (14)$$

$$b_{4m} = -\frac{1}{b_0}\left(\frac{2}{2m+1} b_{4m-3} + b_1 b_{4m-1} + \ldots + b_i b_{4m-i} + \ldots + \frac{1}{2} b_{2m}^2\right)$$

$$b_{4m+1} = -\frac{1}{b_0}\left(\frac{4}{4m+3} b_{4m-2} + b_1 b_{4m} + \ldots + b_1 b_{4m-i+1} + \ldots + b_{2m} b_{2m+1}\right)$$

$$b_{4m+2} = -\frac{1}{b_0}\left(\frac{A\,(2m-1)!!}{2^m (m+1)!} + \frac{1}{m+1} b_{4m-1} + b_1 b_{4m+1} + \ldots + b_i b_{4m-i+2} + \ldots + \frac{1}{2} b_{2m+1}^2\right) \qquad (m = 1, 2, 3, \ldots).$$

Let us now estimate what kind of solution we will get for neutralization of a cesium beam. We compute the quantity λ_2 from

$$\lambda^2 = \frac{2\Pi}{(1-\Phi_* / \Phi_0)^{3/2}} \sqrt{\frac{M}{2e}}\, \frac{M\,(1 + m / M)^{3/2}}{m} \qquad \left(\Pi = \frac{eJ_i}{\Phi_0^{3/2}}\right)$$

Here Π is the system perveance; this quantity is of the order $10^{-8} - 10^{-9}$ a/V$^{3/2}$ for cases of interest [7].

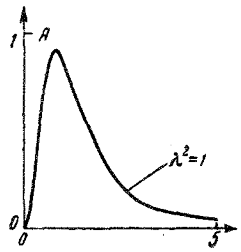

Fig. 3

If we assume that $\Phi_* \ll \Phi_0$, then $\lambda_2 \sim 10^3$ for neutralization by electrons, i.e., we get damped potential oscillations. However, if the same beam is neutralized by negative ions, beginning with hydrogen ions, then the potential does not oscillate, but monotonely increases to its limiting value.

Figure 2 gives the result of numerical calculation for the case of a cesium beam neutralized electrons ($\alpha = 2.71 \cdot 10^{-3}$, $A = 2.98 \cdot 10^{-3}$).

In the general case, the effect of the characteristic parameters A and α on the form of the solution for the potential is shown in Fig. 3. The region lying below the curve corresponds to monotonic behavior of the potential, while the region above the curve corresponds to damped oscillations.

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